Optimal PID Controller Tuning Method for Single-Input/Single-Output Processes

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Introduction

Numerous proportional-integral-derivative (PID) controller tuning methods have been proposed since the ZN tuning rule (Ziegler and Nichols, 1942). Examples are the Cohen and Coon method for the first-order plus time delay model, and modified ZN rules to improve the ZN rule. Also, direct synthesis methods like the Dahlin's and Vogel-Edgar algorithm for digital applications have been proposed (Seborg et al., 1989). The internal model control (IMC) tuning method permits the PID controller to achieve the same control performance with the IMC controller (Morari and Zafiriou, 1989; Rivera et al., 1986). The ITAE-1 tuning rule for the first-order plus time delay model and the ITAE-2 tuning rule for the second-order plus time delay model were designed to obtain tuning parameters that minimize the integral of the time-weighted absolute value of the error (ITAE) criterion (Lopez et al., 1967; Sung et al. 1996). Extension of the ITAE-1 and the ITAE-2 rule for more general processes can be done with a model reduction step (Sung and Lee, 1996). Also, several authors developed tuning strategies that use a pre-determined desired trajectory (Lee et al., 1998). De Paor and O'Malley (1989) proposed a new ZN tuning rule for unstable processes. Kwak et al. (1997) and Kwak et al. (2000) used a two-degree-of-freedom PID controller to remove inherent limitations of the PID controller in controlling integrating or unstable processes and proposed the corresponding tuning rules. For the automatic tuning of PID controllers, Åström and Hägglund (1984) used relay feedback and proposed phase/gain margin specification tuning methods.

These tuning methods have contributed much to improve the control performances of PID controllers. Nevertheless, much room still remains to enhance the PID tuning strategy: (1) Frequently, set points or disturbances are not step signals in real plants. In this case, previous tuning approaches do not give the optimal solution because they have been usually developed to solve step set point tracking or step disturbance rejection problems. (2) Sometimes, the sampling time cannot be reduced enough to guarantee continuous-time control systems due to various reasons like limitations of measurement equipments, heavy computation load, and so on. Many previous tuning methods cannot treat the discrete-time PID controller systematically since they were mainly designed for continuous-time PID controllers. (3) Tuning methods like the ITAE-1, ITAE-2, IMC, and so on cannot be applied directly to more general processes since they were proposed for the first/second order plus time delay model. (4) Almost all previous methods do not guarantee the optimal solution. (5) Industrial PID controllers have various forms. For examples, anti-derivative-kick PID controllers have been used in industry and a lowpass filter is placed after the ideal PID controller. Previous tuning methods cannot treat systematically the structural differences between industrial PID controllers and the ideal PID controller.

We recommend an optimal PID controller tuning strategy that uses the Levenberg-Marquardt optimization method with analytical derivative formulas to solve the above-mentioned problems. It can systematically incorporate various processes, various industrial PID controllers as well as various disturbance/set point shapes with securing optimal control performances in terms of the criterion of the time-weighted integral of the square error.

Proposed PID Tuning Method

Process model

In this research, the following single-input-single-output (SISO) process model will be considered

$$\frac{dx(t)}{dt} = Ax(t) + Bu(t - T_d) + Kd_s(t) \tag{1}$$

$$y(t) = Cx(t) + d_o(t) \tag{2}$$

where $u(t)$ and $y(t)$ denote the process input and the process output, respectively, and T_d is the input time delay. $x(t)$ is the n-dimensional state vector. $d_s(t)$ and $d_o(t)$ are the state disturbance and the output disturbance, respectively. A, B, K

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and C are the following respective forms

$$A = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & -a_n \\ 1 & 0 & 0 & \cdots & 0 & -a_{n-1} \\ 0 & 1 & 0 & \cdots & 0 & -a_{n-2} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & -a_2 \\ 0 & 0 & 0 & \cdots & 1 & -a_1 \end{bmatrix} \quad (3)$$

$$B = \begin{bmatrix} b_n & b_{n-1} & b_{n-2} & \cdots & b_2 & b_1 \end{bmatrix}^T \quad (4)$$

$$K = \begin{bmatrix} k_n & k_{n-1} & k_{n-2} & \cdots & k_2 & k_1 \end{bmatrix}^T \quad (5)$$

$$C = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & 1 \end{bmatrix} \quad (6)$$

Here, Eqs. 1 and 2 can be rewritten equivalently like the following continuous-time Auto-Regressive, Moving Average with an eXogenous input (ARMAX) process model

$$z^{(n)}(t) + a_1 z^{(n-1)}(t) + \cdots + a_n z(t) = b_1 u^{(n-1)}(t - T_d) + b_2 u^{(n-2)}(t - T_d) + \cdots + b_n u(t - T_d) + k_1 d_s^{(n-1)}(t) + k_2 d_s^{(n-2)}(t) + \cdots + k_n d_s(t)$$

$$y(t) = z(t) + d_o(t) \quad (7)$$

The process treated in this research is the most general form among processes considered by previous tuning methods. We can consider any types of disturbances by assigning $d_s(t)$, $d_o(t)$ and K appropriately. For example, if we choose $d_s(t) = 1$, $d_o(t) = 0$ and $K = B$ in Eq. 7 the process has a step input disturbance. Furthermore, the process can be self-regulating, as well as integrating, unstable, or nonminimum phase ones by choosing A,B properly.

Types of PID controllers

It should be noted that various types of PID controllers like ideal, anti-derivative-kick, discrete-time PID controllers, and PID controllers with lowpass filter have been widely used in industry (Poulin et al., 1996; Seborg et al., 1989). Most previous PID tuning methods cannot systematically incorporate them except the ideal one because they have been developed to manipulate mainly the ideal PID controller of Eq. 8 without any systematic techniques to compensate the differences between the ideal PID controller and other types. So, there is a big loss of performance between the tuning step and the practical implementation if we use previous tuning approaches

$$u(t) = k_p[y_s(t) - y(t)] + k_i \int_0^t [y_s(\tau) - y(\tau)] d\tau + k_d \frac{d[y_s(t) - y(t)]}{dt} \quad (8)$$

Where k_p, k_p/k_i and k_d/k_p are the proportional gain, integral time constant, and derivative time constant, respectively. The proposed tuning method can manipulate directly these various types of PID controllers without adding any complexity or difficulties.

Cost function

The objective of the proposed tuning method is to estimate the tuning parameters of the PID controller minimizing the following time-weighted cost function

$$\min_{k_p,k_i,k_d} \left\{ V(k_p,k_i,k_d) = \frac{0.5}{N\Delta t} \sum_{i=1}^{N} w(t_i)[y_d(t_i) - y(t_i)]^2 \Delta t \right\} \quad (9)$$

where k_p,k_i,k_d and Δt represent the proportional gain, integral gain, derivative gain and the sampling time. $y_d(t)$ and $y(t)$ are the desired trajectory (which can be the set point or pre-specified trajectory by the user) and the model output. We can compromise between the control performance and the robustness by adjusting the time constant of the desired trajectory, as done in IMC and direct synthesis methods and/or by using the worst-case model for given parameter uncertainties, as done by Lee et al. (1999). $w(t)$ is a time-weighting function. In this research, we choose the time-weight of $w(t) = t^p \cdot p$ is a nonnegative real value. If we want to shorten the rising time while allowing the larger overshoot for a step set point change, a small p should be chosen and vice versa.

To solve the optimization problem, we use the following Levenberg-Marquardt optimization method of Eqs. 10 and 11 because its convergence rate is very fast and we can calculate the first, as well as the second, derivatives of the cost function with acceptable accuracy from analytical formulas (which will be derived later) and we can estimate good initial values with previous tuning methods

$$\theta(j) = \theta(j-1) - \left\{ \frac{\partial^2 V[\theta(j-1)]}{\partial \theta^2} + \lambda I \right\}^{-1} \times \left\{ \frac{\partial V[\theta(j-1)]}{\partial \theta} \right\} \quad (10)$$

$$\theta = [k_p k_i k_d]^T \quad (11)$$

Where j denotes the iteration number and λ is a small positive value that can be updated every iteration to compromise between the robustness and the convergence rate (for details, refer to Reklaitis et al. (1983)). We repeat Eq. 10 until the tuning parameters converge.

The derivatives of the cost function with respect to the adjustable parameters can be derived as follows: From Eq. 9, we derive Eq. 12 and 13.

$$\frac{\partial V(\theta)}{\partial \theta} = -\frac{1}{N\Delta t} \sum_{i=1}^{N} w(t_i)[y_d(t_i) - y(t_i)] \frac{\partial y(t_i)}{\partial \theta} \Delta t \quad (12)$$

$$\frac{\partial^2 V(\theta)}{\partial \theta^2} = -\frac{1}{N\Delta t} \sum_{i=1}^{N} w(t_i)[y_d(t_i) - y(t_i)] \frac{\partial^2 y(t_i)}{\partial \theta^2} \Delta t + \frac{1}{N\Delta t} \sum_{i=1}^{N} w(t_i) \left[\frac{\partial y(t_i)}{\partial \theta} \right] \left[\frac{\partial y(t_i)}{\partial \theta} \right]^T \Delta t \quad (13)$$

In the Levenberg-Marquardt optimization, it is important to calculate accurately the first and the second derivative of the model output in Eqs. 12 and 13. We use the analytical formulas derived in the next subsection to guarantee acceptable accuracy.

Remarks. (1) Various optimization methods like direct search (Sung et al., 1996), gradient-based optimization methods, or genetic algorithm can be used also. However, the direct search method and genetic algorithm require heavy computation load. Gradient-based search methods suffer from numerical errors if they use numerical derivatives rather than the analytical formulas.

(2) Negative integral time or negative derivative time cannot be realized in most PID controllers and also cannot be optimal because the control performance becomes poorer or even unstable for the negative values. We recommend setting them zero if they enter negative zone during the optimization. However, this situation nearly does not happen if we appropriately determine the initial settings.

Derivative calculation

We can use the following differential equations to calculate the first derivative values of the model output:

$$\frac{d}{dt}\left(\frac{\partial x(t)}{\partial \alpha}\right) = A\frac{\partial x(t)}{\partial \alpha} + B\frac{\partial u(t-T_d)}{\partial \alpha} \tag{14}$$

$$\frac{\partial y(t)}{\partial \alpha} = C\frac{\partial x(t)}{\partial \alpha} \tag{15}$$

Here, α is one of the tuning parameters, k_p,k_i and k_d. Equations 14 and 15 are derived directly from Eqs. 1 and 2. By the same way, we can obtain the second derivatives too. Also, $\partial u(t-T_d)/\partial\alpha$ in Eq. 14 can be derived easily. For example, the first derivative of the ideal PID controller output with respect to the integral gain is like

$$\frac{\partial u(t)}{\partial k_i} = -k_p\frac{\partial y(t)}{\partial k_i} + \int_0^t (y_s(\tau)-y(\tau))\,d\tau - k_i\int_0^t \left(\frac{\partial y(\tau)}{\partial k_i}\right)d\tau - k_d\frac{d}{dt}\left(\frac{\partial y(t)}{\partial k_i}\right) \tag{16}$$

All necessary derivatives for any types of PID controllers can be obtained in a similar way. Now, we can calculate the derivatives of the model output in Eqs. 12 and 13 by solving Eqs. 14–15 and necessary differential equations for the second derivatives using ordinary differential equation solvers like the Runge-Kutta method. Initial values of $\partial x(t)/\partial\alpha$ and $\partial^2 x(t)/\partial\alpha\partial\beta$ are zeros since the tuning parameters do not change the initial values of $x(0)$. We can then calculate the derivatives of the cost function with respect to the tuning parameters. Note that the derived derivative formulas are exact and the related ordinary differential equations can be solved efficiently using various ODE solvers. So we can guarantee acceptable accuracy in calculating the first and second derivatives of the cost function.

Table 1. Parameter Convergence of the Proposed Method

Inter.	k_p	k_p/k_i	k_d/k_p	$V(\theta)$	$\partial V/\partial\theta$	λ
0	2.019	1.228	0.840	0.673	(−1.067, 0.769, 0.410)	—
1	1.973	2.298	0.677	0.191	(−0.150, −0.001, 0.026)	0.5
2	2.249	2.452	0.639	0.159	(−0.088, −0.007, −0.015)	0.5/2
3	2.606	2.648	0.621	0.130	(−0.057, −0.001, −0.018)	0.5/4
3	3.047	2.885	0.609	0.106	(−0.036, 0.004, −0.013)	0.5/8
4	3.549	3.155	0.599	0.090	(−0.021, 0.006, −0.008)	0.5/16
5	4.024	3.420	0.593	0.081	(−0.009, 0.004, −0.004)	0.5/32
6	4.301	3.592	0.592	0.079	(−0.002, 0.001, −0.001)	0.5/64
7	4.369	3.637	0.593	0.079	(−0.000, 0.000, −0.000)	0.5/128

Simulation Study

Consider the following second-order plus time delay process of Eq. 17 controlled by the anti-derivative-kick PID controller of Eq. 18.

$$y(s) = \frac{\exp(-0.3s)}{s^2+1.2s+1}u(s) \tag{17}$$

$$u(t) = k_p[y_s(t)-y(t)] + k_i\int_0^t [y_s(\tau)-y(\tau)]\,d\tau - k_d\frac{dy(t)}{dt} \tag{18}$$

We try several previous tuning (ITAE-2, IMC, ZN) methods and the proposed method to tune the controller. We use the Runge-Kutta method to solve the differential equations with 0.01 sampling time. The time weight of $w(t)=t^2$ and the tuning parameters of the ITAE-2 method as the initial settings are chosen for the proposed tuning method. Table 1 shows stable and fast parameter convergence of the proposed method. The computation load is definitely tolerable when considering the computing power of today. The previous tuning methods show unacceptable control performances for the anti-derivative-kick controller, as shown in Figure 1 because they are not originally designed for the anti-derivative-kick

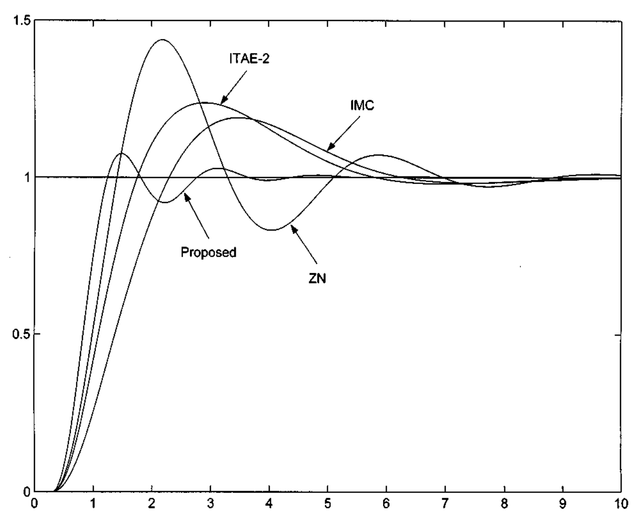

Figure 1. Tuning of this method and previous (ITAE-2, IMC, ZN) methods for antiderivative-kick PID controller.

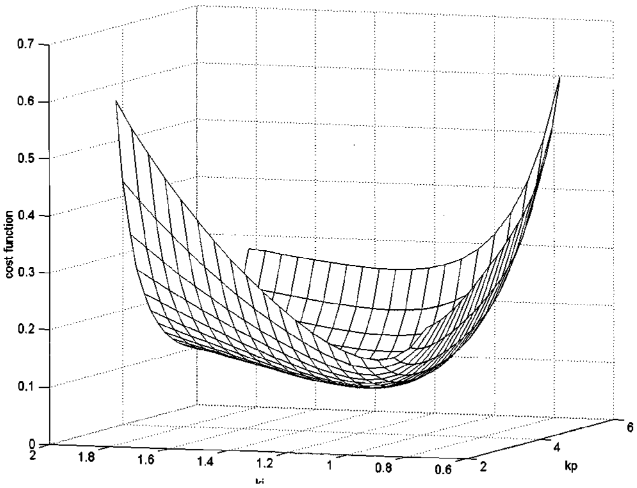

Figure 2. Typical geometry of the cost function in this method.

PID controller. For the IMC tuning method (Rivera et al., 1986), the time delay in Eq. 17 is replaced by the first-order Taylor series approximation and the time constant of the first-order IMC filter is set by 0.7. A smaller time constant of the IMC filter shortens the rising time, but the overshoot increases. The proposed method provides the optimal tuning because it can directly manipulate the anti-derivative-kick PID controller. We did check the three-dimensional (3-D) geometry plots of the cost function for various combinations of the adjustable parameters and found no symptoms of local minima and unacceptably distorted contour. Figure 2 shows a typical one. We did extensive simulation studies and recognized that the proposed method can directly manipulate various disturbances/set points and various processes/controllers without adding any complexity or difficulties. We did not include the results due to space limitation.

Conclusions

We recommend an optimal tuning strategy for PID controllers to incorporate various situations in industry. The computation load is tolerable since the proposed method uses the Levenberg-Marquardt optimization strategy with analytical derivative formulas. The proposed tuning method is much more versatile than previous tuning methods. It can secure the optimal solution for the criterion of the time-weighted integral of the square error. Also, various processes and various PID controllers can be incorporated. Furthermore, it can manipulate various set point changes/disturbances in an optimal manner.

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